MIT | Arts, Commerce & Science College

Department of Computer Science
Class: F.Y.B.Sc. (Comp. Sci.)
Mathematics Question Bank

Sub: Discrete Mathematics

Ch.1 Logic

2 Marks

1. Translate into symbolic form:
   a) All juniors are clever.
   b) Some men do not like cats.
2. Write the following statement in symbolic form and hence write its negation.
   'Every FYBCS student studies Discrete Mathematics.'
3. Determine the truth set of the following proposition over positive integers:
   a) p(n): n is perfect square and n<100.
   b) q(n): n is prime and n<25.
4. Obtain the negation of each of the following:
   a) $\forall x \in \mathbb{R}\left(x^2 - 10 \leq 18\right)$
   b) $\forall x, \exists y\left[p(\mathrm{x}) \wedge \mathrm{q(y)}\right]$

4 Marks

1. Show that $p \rightarrow (\mathrm{q} \rightarrow \mathrm{r})$ and $p \rightarrow (\sim q \vee r)$ are logically equivalent.
2. Check the following logical equivalence using truth table:
   $\left[a \rightarrow (\mathrm{a} \wedge \mathrm{c})\right] \equiv \left[(\mathrm{a} \rightarrow \mathrm{b}) \wedge (\mathrm{a} \rightarrow \mathrm{c})\right]$
3. Convert the following arguments in symbolic form and hence write its converse, inverse and contra positive.
   a) If I am not President of India then I will walk to work.
   b) A positive integer is a prime only if it has no divisors other than 1 & itself.
   c) The home team wins whenever it is raining.
4. Verify for tautology & contradiction:
   a) $(p \Lambda q)\ \Lambda \sim (p \vee q)$
   b) $(a \Lambda b) \vee \sim (a \wedge b)$
   c) $\left[(p \wedge q) \rightarrow r\right] \rightarrow \left[p \rightarrow (q \rightarrow r)\right]$
5. Test the validity of the following arguments by direct method.
   a) $p \rightarrow \sim q, \sim r \rightarrow p, q \mapsto r$

b) $p, p \to q, s \vee r, r \to \sim q \mapsto s \vee t$

c) $p \to q, r \to \sim t, p \vee r \mapsto q$

d) $r \to c, s \to \sim w, r \vee s, w \mapsto c$

e) $d \vee \sim e, \sim e \to f, d \to \sim g \mapsto \sim g \vee f$

f) $r \to p, g \to m, p \vee m \to s, \sim s \mapsto \sim (r \vee g)$

6. Test the validity of the following arguments by indirect method.

a) $\sim p \vee q, s \vee p, \sim q \mapsto s$

b) $r \to c, s \to \sim w, r \vee s, w \mapsto c$

c) $A \vee \sim B, \sim C \to B, A \to \sim D \mapsto \sim (D \vee C)$

d) $p \vee \sim q, r \to \sim q, q \mapsto \sim r$

7. Translate the following into simple English if, $c(\mathrm{x})$: x is a comedian and $f(\mathrm{x})$: x is funny. The domain is the set of all people.

a) $\forall \mathrm{x}\left(c(\mathrm{x}) \to \mathrm{f}(\mathrm{x})\right)$

b) $\forall \mathrm{x}\left(c(\mathrm{x}) \wedge \mathrm{f}(\mathrm{x})\right)$

c) $\exists \mathrm{x}\left(c(\mathrm{x}) \to \mathrm{f}(\mathrm{x})\right)$

d) $\exists \mathrm{x}\left(c(\mathrm{x}) \wedge \mathrm{f}(\mathrm{x})\right)$

8. Write the following argument in symbolic form and test the validity.

a) Team A will win the cricket match if and only if they are playing against B. If team A does not win, then team C will take away the trophy. Team C does not get the trophy. Hence team A does not play against team B.

b) If Ravi knows Pascal, then he passes the examination. Ravi knows Java. If Ravi knows Java, then he is selected for campus interview or he does not pass the examination. Therefore Ravi does not know Pascal. (p, q, r, s)

c) If Rani's family shifts to Pune she will be admitted to Engineering college. If her family shifts to Mumbai, she will be admitted to Medical college. If she goes to Engineering or medical she will definitely be settled in life. But Rani is not settled. Hence Rani is neither in Pune nor in Mumbai.

Ch.2 Lattices and Boolean algebra

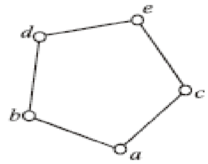
2 Marks

1. Draw Hasse diagram for the relation 'divides' on the set {1, 2, 3, 4, 6, 12}.

4 Marks

1. Let A = {1, 3, 5, 7, 9} be any set. Define a relation R on A as "x is related to y" if and only if $x \leq y$. Is A a poset with respect to R? Justify.
2. Draw the Hasse diagram for the poset D_{20} with respect to the "divisibility" relation. Is it complemented lattice? Justify.

3. Let $A=\{a,b\}, and\ B=P(A)$ be the power set of A. Write the operation tables for join and meet on B .
4. Define distributive lattice. Is the following lattice distributive? Justify.



5. Find the complement of each element of the lattice D_{24} with respect to the "divisibility" relation.
6. Write disjunctive normal form and conjunctive normal form of the following Boolean functions.
   a) $f(\text{x},\text{y},\text{z}) = \text{y} \vee \left(\overline{x} \wedge z\right)$
7. Write the disjunctive normal form of the Boolean function
   a) $f(x, y) = \overline{x} \vee y$
8. Determine whether the poset D_{125} with respect to the "divisibility "relation is a Boolean algebra.

8 Marks

1. Draw Hasse diagram for D_{30}-positive divisors of 30 with the partial order 'divides'. Find maximal and minimal elements. Also find complement of each element. List the atoms of this lattice.
2. Draw Hasse diagram for D_{12}-positive divisors of 12 with the partial order 'divides'. Write join and meet tables for this poset. Determine whether it is Boolean Algebra.
3. Give an example of each of the following with justification:
   a) A poset which is not a lattice.
   b) A non-distributive lattice.
   c) A lattice which is not complemented.
   d) A distributive lattice which is not complemented.
   e) A Boolean algebra.
4. Write the following Boolean expression in disjunctive normal form. $f(\text{x},\text{y},\text{z}) = \text{y}.(\overline{x+z}) + (\text{x}\,.\text{y} + \overline{z}.\text{x})$
5. Let $f(x_1, x_2, x_3) = (\text{x}_1 \Lambda x_2) \vee (\text{x}_1 \Lambda \text{x}_3) \vee (\overline{x}_1 \Lambda x_3)$ be a Boolean expression. Write $f(\text{x}_1, \text{x}_2, \text{x}_3)$ in conjunctive normal form.
6. If $f(x_1, x_2, x_3) = (\overline{\text{x}}_3 \Lambda x_2) \vee (\overline{\text{x}}_1 \Lambda \text{x}_3) \vee (x_2 \Lambda x_3)$ is a Boolean function over the two valued Boolean algebra, write $f(\text{x}_1, \text{x}_2, \text{x}_3)$ in disjunctive normal form.

Ch.3 Counting Principles

2 Marks

1. Let A and B be two sets. Suppose |A|=3 and |B|=4. Find the number of functions from A to B.
2. How many arrangements of the letters of the word “ACCEPTANCE” are there?
3. In how many ways can the letters in the following word “MATHEMATICS” be arranged?

4 Marks

1. How many integers between 1 and 1000 are divisible by 2 or 5 or 7?
2. A committee of five is to be selected among 6 boys and 5 girls. Determine the number of ways of selecting the committee, if it is to consist of at least one boy and one girl.
3. How many four digit numbers are there that begin with 3 or end with 3?
4. How many elements are there in the union of four sets, if the sets have 50, 60, 70 and 80 elements respectively, each pair of the sets has 5 elements in common, each triple of the sets has 1 common element and no common element in all 4 sets?
5. There are 9 faculty in the mathematics department and 11 in computer science department. How many ways are there to form a committee of 7 members to develop a course, If the committee is to consists of at least 3 and at most 5 members from the mathematics department?
6. Show that if 5 integers are selected from the first eight positive integers, then there must be a pair with a sum equal t 9.
7. Show that if 21 integers are chosen from the numbers 1 to 40, then there is one integer which divides the other.
8. Show that at a party of 20 people, there are two people who have the same number of friends.
9. What is the minimum number of students required in Maths class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D, E?

Ch.4 Recurrence Relation

2 Marks

1. What are the characteristic roots of the recurrence relation $a_r - 7a_{r-1} + 12a_{r-2} = 0$?
2. Find first four terms of the sequence defined by the following recurrence relation: $a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 1, a_1 = 2$.

4 Marks

1. Solve the following homogeneous recurrence relations.
   a) $a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$, $a_0 = 0, a_1 = 2, a_2 = 8$.
   b) $a_n - 7a_{n-2} + 6a_{n-3}$, $a_0 = 8$, $a_1 = 6$, $a_2 = 22$
   c) $a_n = 10a_{n-1} - 32a_{n-2} + 32a_{n-3}$, $a_0 = 5, a_1 = 18, a_2 = 76$
   d) $a_n = 3a_{n-1} + 4a_{n-2}$ for $n \geq 2$; $a_0 = a_1 = 1$

e) $a_n = 6a_{n-1} - 8a_{n-2};\ a_0 = 1, a_1 = 3$

f) $a_n - 12a_{n-2} + 16a_{n-3} = 0; a_0 = 4, a_1 = -8, a_2 = -12$

8 Marks

1. Solve the following non homogeneous recurrence relations.

   a) $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r,\ \ a_0 = 1,\ a_1 = 1.$

   b) $a_n - 2a_{n-1} + a_{n-2} = 3 - 4n,\ \ a_0 = a_1 = 1$

   c) $a_n - 6a_{n-1} + 9a_{n-2} = 3^{n+1},\ \ a_0 = 0, a_1 = 1$

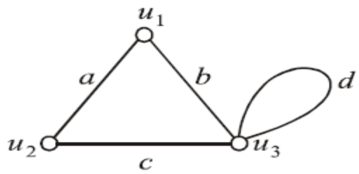
   d) $a_n + a_{n-1} - 6a_{n-2} = (2 + 6n)5^n,\ \ a_0 = 1, a_1 = 2$

   e) $a_{n+3} + 2a_{n+2} - 5a_{n+1} - 6a_n = 32;$

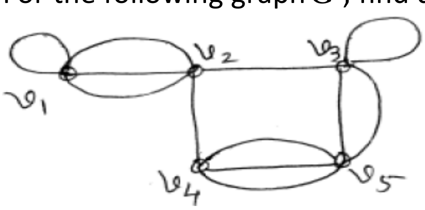
   f) $a_n = 2a_{n-1} + 3^n;\ a_1 = 5$

Ch.5 Graph

1. Verify Handshaking lemma for the following graph.



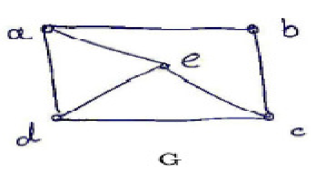
2. For the following graph G, find degree of each vertex and verify Handshaking Lemma.



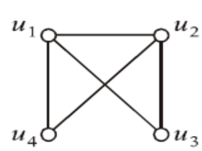
3. Show that the maximum number of edges in simple graph with n vertices is $\dfrac{n(n-1)}{2}$.

4. Write adjacency and incidence matrix of the following graphs.

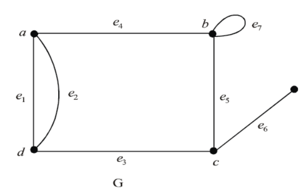
   a)



   b)



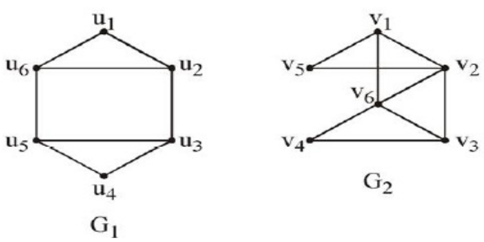
   c)



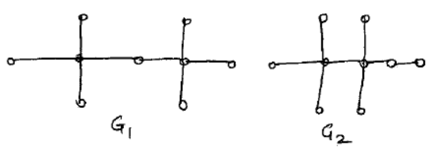
5. Draw the graphs represented by adjacency matrix and incidence matrix.

$$A(\mathrm{G})=\begin{bmatrix}0&3&0&0\\3&0&1&1\\0&0&2&2\\0&1&2&0\end{bmatrix},\qquad I(\mathrm{G})=\begin{bmatrix}1&0&0&0&1\\1&1&1&0&0\\0&0&1&1&0\\0&1&0&1&1\end{bmatrix}$$

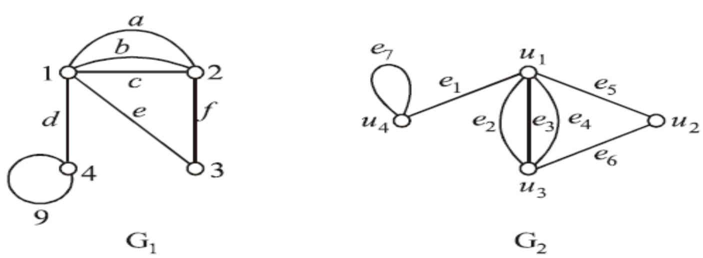
6. Define each of the following with example.
   I. Complete graph
   II. Bipartite graph
   III. Complete bipartite graph
   IV. Regular graph
7. Determine whether the following graphs are isomorphic or not.
   a)



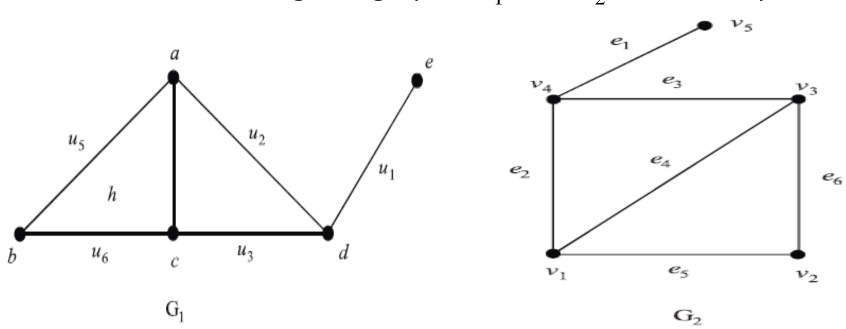
   b)



8. Show that the following two graphs G_1 and G_2 are isomorphic.



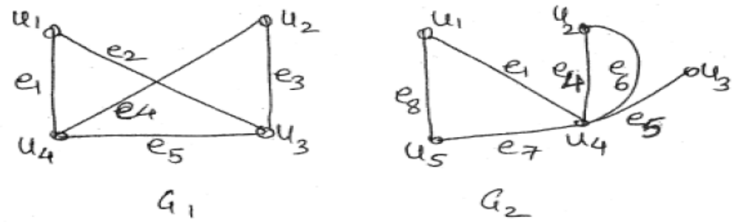
9. Show that the following two graphs G_1 and G_2 are isomorphic.



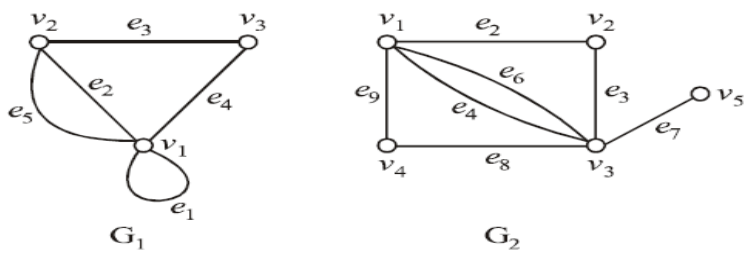
10. Draw all non isomorphic simple graphs on 3 vertices.
11. Find the minimum number of vertices in simple graph with at least 25 edges.

Ch.6 Operations on Graph

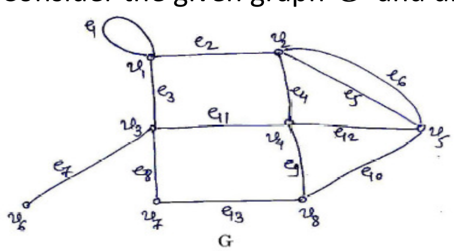
1. Find union, intersection and ring sum of the following graphs.



2. Find $G_1 \cup G_2$ and $G_1 \oplus G_2$ of the following graphs.



3. Consider the given graph G and draw the following subgraphs



i. $G - A,\ A = \{v_1, v_4, v_5\}$

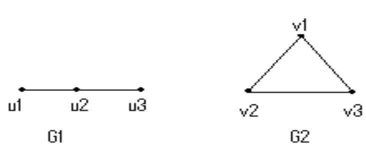
ii. $G - F,\ F = \{e_3, e_9, e_{12}\}$

iii. $G\langle F\rangle,\ F = \{v_1, v_2, v_3\}$

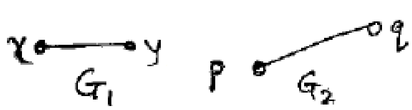
iv. $G\langle F\rangle,\ F = \{e_7, e_{11}, e_8, e_{13}\}$

4. Find $G_1 \times G_2$ for the following graphs.

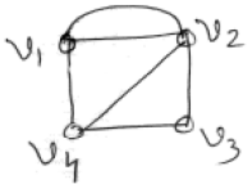
a)



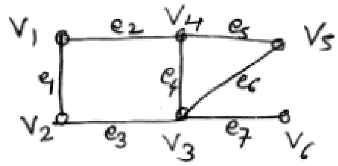
b)



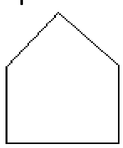
5. Fuse the vertices v_1 and v_3 of the following graph and draw the resulting graph.



6. Fuse vertices v_3 and v_4 in the following graph.

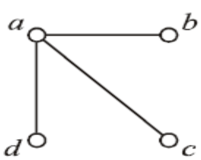


7. Find complement of the following graph and determine whether it is self complementary or not.

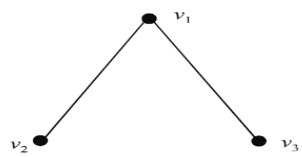


8. Find the complement of the following graph.

a)



b)

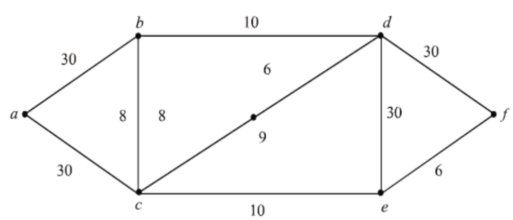


9. If G is self complementary graph on n vertices, then show that n is of the form $4k$ or $4k+1$ for some integer k.

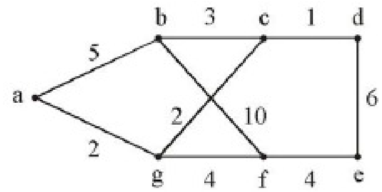
Ch. 7 Connected Graph

1. Determine the shortest path from vertex a to all other vertices in the following weighted graphs using Dijkstra's Algorithm.

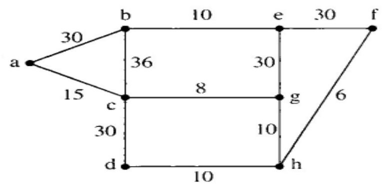
a.



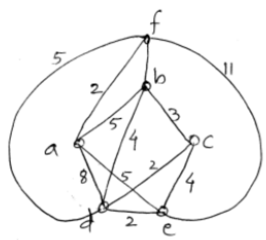
b.



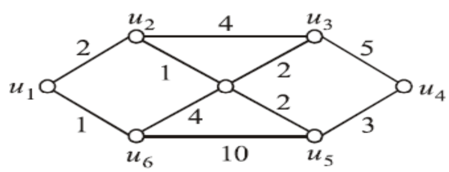
c.



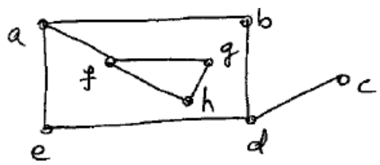
d.



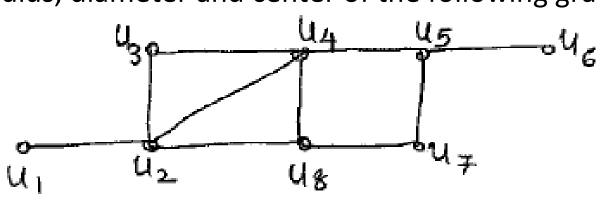
2. Using Dijkstra's Algorithm, find shortest path between u_1 and u_4.



3. Find all cut vertices and isthmuses in the following graph.

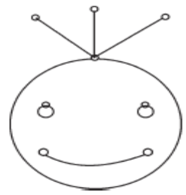


4. Find radius, diameter and center of the following graph.



5. Define i) Vertex connectivity ii) Edge connectivity
   Draw complete bipartite graph $K_{4,3}$. Also find its vertex connectivity and edge connectivity.
6. Give an example of graph with vertex connectivity $K(G) = 4$ and edge connectivity $\lambda(G) = 5$. Justify.

7. Find the number of components in the following graph.



8. Read all the theorems and their proof in this chapter (Connected Graph).

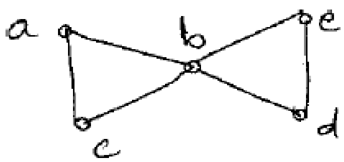
Ch. 8 Eulerian and Hamiltonian Graph

1. Define i) Konigsberg Bridge Problem
   ii) Eulerian graph
   iii) Chinese Postmen Problem

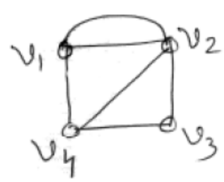
2. Define i) Hamiltonian graph
   Ii) Traveling Salesmen Problem

3. Determine whether the following graphs are Eulerian or not. Justify.

a)

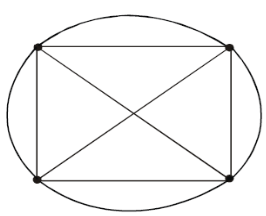


b)



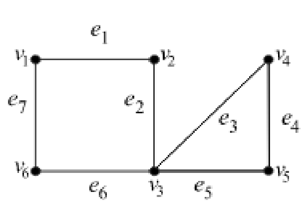
c) $K_{3,4}$

d)

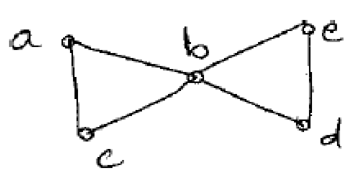


4. Using Fleury's Algorithm find Eulerian tour in the following Eulerian graphs.

a)

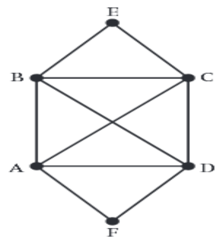


b)

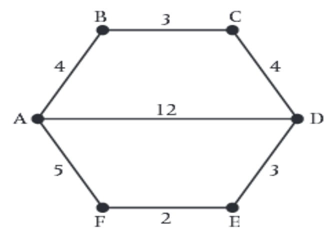


5. Solve the Chinese Postmen Problem for the following graphs.

   a)

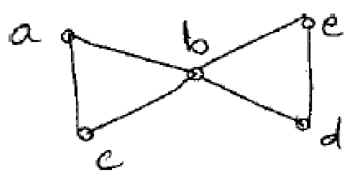


   b)

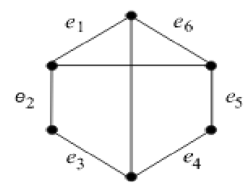


   b) Determine whether the following graphs are Hamiltonian or not. Justify.

   a)



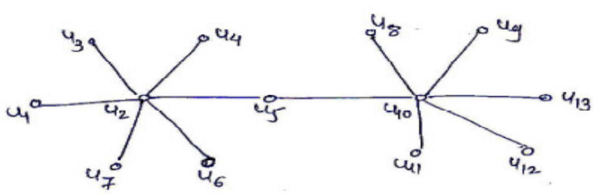
   b)



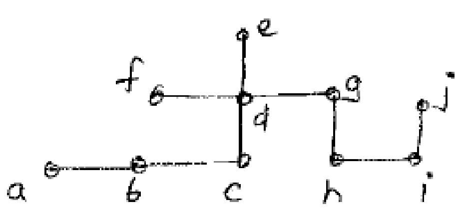
Ch. 9 Trees

1. Find eccentricity of all the vertices and hence find radius, diameter and center of the following Trees.

   a)

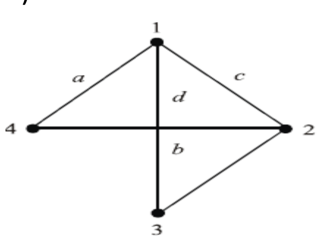


   b)

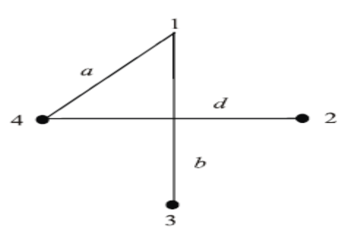


2. Find all fundamental cut sets and Fundamental cut circuits in the following graphs with respect to given spanning tree.

   a)

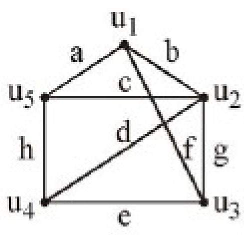


G

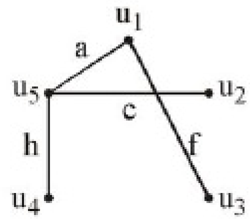


T

b)

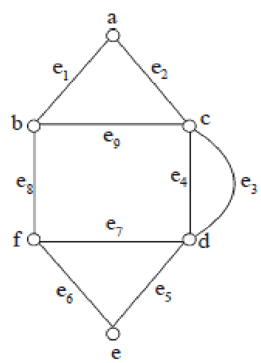


G

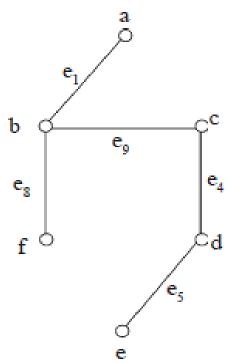


T

c)



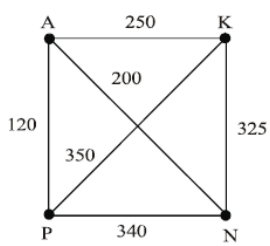
(G)



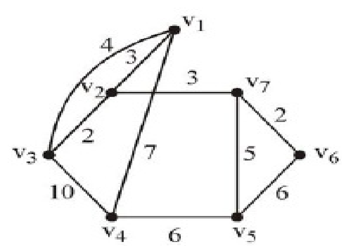
(T)

3. Using Kruskal's Algorithm, find shortest spanning tree of the following weighted connected graphs.

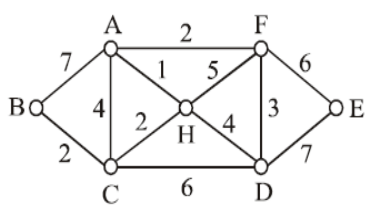
a)



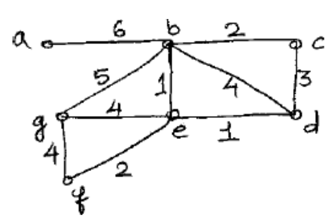
b)



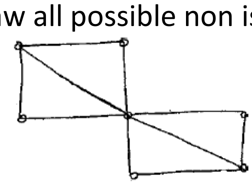
c)



d)



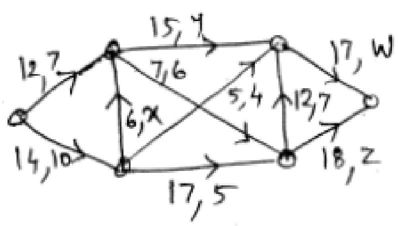
4. Draw all possible non isomorphic spanning trees of the following graph.



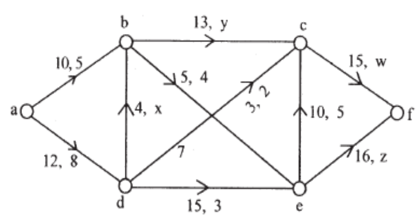
5. Draw a binary tree on 25 vertices with minimum and maximum height.
6. Draw a binary tree on 31 vertices with minimum and maximum height.
7. Find Preorder, In order and post order traversal for any rooted tree.
8. Draw the arborescence and find prefix (Polish) notation of the following expressions.
   a) $\dfrac{3x+y}{(6a-3b)^7}$
   b) $(5x+8)(7y^3-2)^7$
   c) $(x+y)^4+\dfrac{x-4}{3}$
9. Find the prefix form of the expression $\left(x+\dfrac{y}{z}\right)+3$.
10. Simplify the following prefix notations and draw the arborescence.
    a) $+ - * \; 2 \; 3 \; 5 \; / \uparrow 2 \; 3 \; 4$
    b) $* + 3 + 3 \uparrow 3 + 3 \; 3 \; 3$
11. Study all the properties of tree.

Ch. 10 Directed Graph

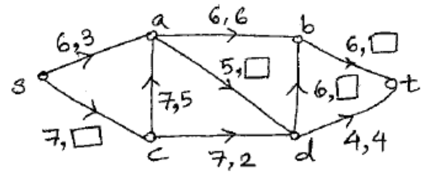
1. Define
   a) Simple symmetric digraph
   b) Complete digraph
   c) Balance digraph
2. Find the values of x, y, z and w in the following network.
   a)



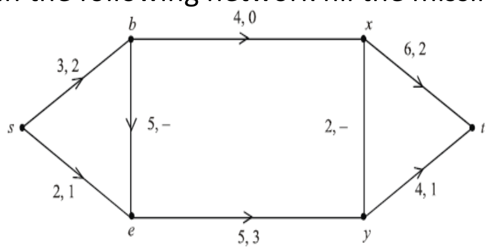
   b)



3. In the following network, fill the blocks with suitable numbers so that the second set of numbers determines a flow in the network.



4. In the following network fill the missing figure. Also find the value of the flow.



Sub: Algebra and Calculus

CHAPTER 1: RELATIONS AND FUNCTIONS

2 marks

1. Define symmetric relation. Give one example.
2. Draw diagraph for the following relation:
   R= {(1, 2), (2, 3), (3, 2), (3, 3)}
3. Let A be set of all straight lines in the plane. Define a relation R on A by LRM iff L is perpendicular to M. Check whether R is reflexive, symmetric and transitive.
4. If:
   B = {x, y, z}
   Write the relation R whose matrix is:
   $$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
5. Define partial order relation. Give an example of partial order relation.
6. List all partition of a set A = {a, b, c} and hence all possible equivalence relations on A.
7. Give an example of a relation which is reflexive, transitive but not symmetric. Justify.
8. Let A = {a, b, c, d}. Determine whether following relation R on A is transitive or not, where R = {(a, a), (a, b), (b, c)}. Justify.
9. Check whether relation
   R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}
   Is transitive on the set A = {1, 2, 3} or not. Justify?
10. Give an example of a relation on the set A = {a, b, c} which is symmetric but not transitive. Justify your answer.

4 marks

1. Let R be the relation
   {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}
   and S be the relation
   {(2, 1), (3, 1), (3, 2), (4, 2)}
   Find the composite relation S°R on the set
   A = {1, 2, 3, 4}
2. Using Warshall's algorithm, obtain transitive closure of the following relation:
   R = {(1, 2), (2, 2), (2, 4), (3, 2), (3, 4), (4, 1)}
3. Define an equivalence relation. Let R be an equivalence relation on set X. Show that for any x, y $\in$ X, x $\in$ [y] iff [x] = [y].
4. Let A = {1, 2, 3} and R = {(1, 1), (1, 2), (1, 3), (2, 2), (3, 1), (3, 3)} be a relation on set A. Identify if R is reflexive, symmetric, transitive. Justify your answer.

5. Draw diagraph of relation R given by aRb iff a and b are not relatively prime. a, b ∈ A where A = {1, 2, 3, 4, 5} .
6. Define partition of a set and prove if S is a partition of set A, then there exists an equivalence relation on set S such that equivalence classes will form partition of A namely S.
7. Prove that any two equivalence classes are either disjoint or identical.
8. Let R be the relation on the set of ordered pairs of positive integers defined as $(a,b)R(c,d)$ if and only if $a+d=b+c$. Also interpret geometrically the equivalence classes of the points P (2, 7) and Q (-1, 3).
9. Let a function $f:R\to R$ is defined as $f(x)=2x-3$. Show that f is bijective.
10. Let $f:R\to R$ be defined by:

$$f(x)=\frac{5x+11}{3}$$

Show that f is bijective. Find f^{-1}.

11. Give an example of each of the following with justification. A relation which is
    a) Reflexive, symmetric but not transitive.
    b) Reflexive, transitive but not symmetric.
    c) Transitive, symmetric but not reflexive.
    d) Symmetric as well as antisymmetric.
    e) Neither reflexive nor symmetric but transitive.

8 marks

1. Let ~ be an equivalence relation on a non – empty set A, then prove that:
   (i) $a\in[a], \forall a\in A$.
   (ii) Any two equivalence classes are either disjoint or identical.
2. Let A = {1, 2, 3, 4}, R = {(1, 4), (2, 2), (2, 3), (3, 2), (4, 3)}. Find transitive closure of R denoted by R^*. Also find digraph of R^*.
3. Let A = {1, 2, 3, 4, 5} and R = {(1, 1), (1, 4), (2, 2), (3, 4), (3, 5), (4, 1), (5, 2), (5, 5)} be a relation on A. Find transitive closure of R. Also find digraph of transitive closure of R.
4. Let R = {(1, 1), (1, 3), (2, 2), (2, 4), (3, 3), (3, 1), (4, 4), (4, 2)} be the relation on A={1,2,3,4}. Show that R is an equivalence relation on A. Also write the equivalence classes with respect to relation R.
5. Write down the transitive closure of R = {(a, b), (b, b), (b, c), (c, a), (c, b)}, a relation defined on set A = {a, b, c}, by using Warshall’s algorithm.

CHAPTER 2 : BINARY OPERATIONS AND GROUPS

2 marks

1. Consider the set A = {1, -1, i, -i} with usual multiplication. Write inverse of each element.

2. On Z, the set of integers, define * as $a * b = a^b$. Justify whether * is binary operation or not.
3. List all generators of cyclic group Z_{12} under addition modulo 12.
4. Is every semigroup a monoid? Justify.
5. Let Q+ be set of all positive rational numbers. Define operation * as:
$$a * b = \frac{ab}{3}.$$
Find identity element for this operation.

4 marks

1. Express the following permutation on S_9 as a product of disjoint cycles and hence find order of σ.
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 4 & 3 & 6 & 7 & 5 & 9 & 8 \end{pmatrix}$$
Also determine whether σ is even or odd.
2. Write composition table for (Z_6, +). Is it a cyclic group? If yes, write all generators.
3. Show that the set $S = \{a + b\sqrt{2} \mid a, b \in Z\}$ is a group under addition.
4. Let G = {1, -1, i, -i} be a group under complex multiplication. Obtain order of each element of G where $i = \sqrt{-1}$.
5. Let α and β are two permutations in S_5, such that
$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 3 & 5 \end{pmatrix}$$
Find:
(i) $\alpha \circ \beta$
(ii) $\beta \circ \alpha$
(iii) β^{-1}
(iv) Check whether α is even or odd.
(v) Find orders of α and β.
6. Let Z be set of integers. Define an operation * on z as:
$$a * b = a + b - 5.$$
Show that Z is an abelian group w.r.t. *.

8 marks

1. Show that the set :
$$G = \{\begin{bmatrix} x & x \\ x & x \end{bmatrix} / \, x \neq 0, x \in R\}$$
is an abelian group under usual matrix multiplication.

CHAPTER 3 : DIVISIBILITY OF INTEGERS

2 marks

1. State first principle of mathematical induction.
2. Define Euler's ϕ function. Find ϕ(100).
3. Construct composition table for (Z_8^*, X_8) where (Z_8^*) is prime residue class set in Z_8.
4. Find the greatest common divisor for 263 and 52.
5. For any two integers a and b having greatest common divisor d, find $\left(\frac{a}{d}, \frac{b}{d}\right)$

4 marks

1. Find the remainder of 3^{97} when divided by 31.
2. If
   a ≡ b(mod n), c ≡ d(mod n),
   then prove that:
   (i) (a + c) ≡ (b + d)(mod n)
   (ii) ac ≡ bd(mod n)
3. Prove that no two integers x and y exist satisfying:
   x + y = 200 and (x, y) = 7
4. Find the remainder when $7^{200} + 11^{800}$ is divided by 101.
5. Show that $5^{10} - 3^{10}$ is divisible by 11.
6. Show 19 is not divisor of $4n^2 + 4$ for any integer n.
7. Find remainder of $(1 + 2 + 3 + \ldots\ldots\ldots\ldots + 100)^{100}$ after dividing by 7.
8. For any a, b, x ∈ Z, show that (a, b) = (a, b + ax).
9. Prove that for any two integers a and b, a Ξ b(mod n) if a and b leaves the same remainder when divided by n.
10. Find gcd of 2210 and 357 and express it in the form $(2210)m + (357)n$.
11. Show that the relation $a \equiv b(\bmod n)$ is an equivalence relation on Z.
12. Find the remainder of 7^{456} when divided by 11.
13. Prove that there are exactly n distinct residue classes modulo n.
14. Using mathematical induction, prove that
    $2^n > n^2 \forall n \geq 4$
15. Find the remainder of $7^{777} + 81$ when divided by 11.
16. Prepare addition table for Z_4 modulo 4.
17. Prove by using mathematical induction:
    $$1^3 + 2^3 + \ldots\ldots\ldots\ldots\ldots + n^3 = \frac{n^2(n+1)^2}{4}, \ \forall n \geq 1.$$
18. Find the remainder when $11^{300} + 17^{700}$ is divided by 11.
19. If c|ab and (b, c)=1, then prove c|a.

8 marks

1. Find greatest common divisor (gcd) of 7260 and 1638. Express it in the form 7260m + 1638n where m and n are integers.
2. Find integers x and y such that:
   (7234, 3456) = 7234x + 3456y
3. Find greatest common divisor (gcd) of 3587 and 1819. Express it in the form 3587m + 1819n where m and n are integers.
4. Let p be a prime number and a, b ∈ Z. Then prove that
   (i) $p \mid ab \Rightarrow p \mid a \;\; or \;\; p \mid b$
   (ii) If $p \nmid a$ then $a^{p-1} \equiv 1 \left(\bmod p\right)$
5. Let a_n be the recursive relation defined by
   $a_n = a_{n-1} + a_{n-2} + a_{n-3} \quad n \geq 3$
   With initial conditions:
   $a_0 = 1, \;\; a_1 = 1, \;\; a_2 = 1$
   (a) Obtain first few terms of recursive relation.
   (b) Prove that all $a_n\,'s$ are odd.
   (c) Prove that $a_n \leq 2^{n-1} \;\; \forall \; n \geq 3$.
6. Find greatest common divisor (gcd) of 6162 and 1213. Express it in the form 6162m + 1213n where m and n are integers.

<u>Miscellaneous Problems</u>:

1) Construct the composition table for ($Z_{10}^{*}, \times_{10}$) where Z_{10}^{*} is prime residue class set in Z_{10} .
2) Find the remainder of $(1+2+3+\cdots\cdots+100)^{100}$ after dividing by 7.
3) Z_{12} is the set of residue classes mod 12. Then find the value of
4) Find the greatest common divisor of 7469 and 2464. Express it in the form 7469m + 2464n. Find the values of m & n.
5) Find the greatest common divisor of 119 & 272.
6) Show that $\sqrt{7}$ is not a rational number.
7) Prove that for any two integers a & b, $a \equiv b(\bmod n)$ if a & b leaves the same remainder when divided by n.
8) Let p be the prime number and $a, b \in Z$. Then prove that $p/ab \Rightarrow p/a or p/b$.
9) Obtain the multiplication table for Z_7 .
10) Find the remainder when $7^{361} + 7^{362}$ is divided by 11.
11) List all the elements in Z_8 which satisfy $x^2 = x$.

12) Prove that :
   (i) $19/(2^{18}-1)$
   (ii) $89/(2^{44}-1)$
   (iii) $97/(2^{48}-1)$
13) Let $n\in N$ and $a,b,c\in Z$. If $ac\equiv bc(\text{mod}n)\&(c,n)=d$ where $n=dw,w\in Z$, then prove that $a\equiv b(\text{mod}\,w)$.
14) Find the remainder when $5^{2009}+185$ is divided by 11.
15) Show that $(53,9999)=1$.
16) Find the remainder of 7^{486} when divided by 13.
17) Find the greatest common divisor of 6162 & 1213. Hence find integers m & n such that d = 6162m + 1213n.
18) Prove that $1+2+2^2+2^3+\cdots+2^6$ is a composite number.
19) Apply the first principle of induction to prove:

$$3+3.5+3.5^2+3.5^3+.......+3.5^n=\frac{3\left(5^{n+1}-1\right)}{4};\ for\,all\ n\leq 0$$

20) Find the g.c.d'd' of -1166 and 1474. Also find integers x and y such that $d=-1166x+1474y$.
21) : Find integers x, y and z satisfying $(198,288,512)=198x+288y+512z$
22) The Fibonacci numbers $f_0,f_1,f_{2,.........}$ are defined $f_0=f_1=1, f_n=f_{n-1}+f_{n-2},\forall n\geq 2.$

   Prove that $f_n\leq\left(\frac{7}{4}\right)^n,\forall n\geq 0.$
23) Let R be a relation defined on Z by aRb if and only if $a\equiv b\left(\text{mod}3\right)$. Obtain the distinct equivalence classes of elements in Z under R.
24) Find the units digit of each of the numbers $7^{100}\ and\ 9^{102}$.
25) Find the remainder when $53^{103}+103^{53}$ is divided by 39.
26) Prove that if 'a' is an odd integer, then $a^2\equiv 1\left(\text{mod}8\right)$.

CHAPTER 4 : CONTINUITY AND MEAN VALUE THEOREM

2 marks

1. State true or false with justification. 'Every continuous function is differentiable'.
2. Discuss the applicability of Lagrange's Mean Value theorem to the function f(x) = x $^{1/3}$, x ∈ [-1, 1].
3. State Rolle's Theorem.

4. Discuss the continuity of f(x), where

$$f(x) = \begin{cases} 2x+1 & \text{if } x \le 1 \\ x^2 & \text{if } 1 < x \le 2 \end{cases}$$

5. Give an example of unbounded continuous function on nonclosed interval with justification.
6. True or False: Rolle's theorem is applicable to the function, $f(x) = |x|, x \in R$. Justify.
7. Discuss the continuity of the function:

$$f(x) = \frac{x^2 - 4}{x+2}, x \neq -2$$
$$-4 \quad , x = -2$$

4 marks

1. Find values of α and β if the function is continuous in (-2, 3) where:

$$f(x) = \begin{cases} 4x+5 & \text{if } -2 < x < 0 \\ 2x + \alpha & \text{if } 0 \le x < 1 \\ x - 3\beta & \text{if } 1 \le x < 3 \end{cases}$$

2. Apply Rolle's theorem to the function:

   f(x) = (4 – x) log x

   and show that

   c log c = 4 – c

   for some c ∈ (1, 4)

3. Verify Lagrange's mean value theorem and find c if possible for the function:
   $f(x) = x - x^3$, on [-2, 1].
4. Show that $\sqrt{5}$ is not a rational number.
5. Test the continuity of the following function at the origin

$$f(x) = \begin{cases} \dfrac{x - |x|}{x}, x \neq 0 \end{cases}$$
$$= 0, x = 0$$

6. Using LMVT for the function $f(x) = \sin x + \cos x$ on $\left[0, \frac{\pi}{2}\right]$
7. Verify LMVT and find c if possible for the function $f(x) = x - x^3$ on [-2, 1].

8. Discuss the continuity of the function f(x) defined by

$$f(x)=\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}},\quad x\neq 0$$
$$0 \qquad , \quad x=0.$$

9. Considering the functions:

$$f(x)=\sqrt{x}\quad and\quad g(x)=\frac{1}{\sqrt{x}}$$

Prove that 'c' of Cauchy's mean value theorem is the geometric mean of a and b.

10. Discuss the continuity of f(x) where:

$$\begin{aligned} f(x) &= 2x-1, && x\leq 0\\ &=x^2, && 1<x<2\\ &=3x-4, && 2\leq x<4\\ &=x^{3/2}, && x\geq 4 \end{aligned}$$

8 marks

1. State and prove Rolle's Mean Value Theorem and hence explain the geometric interpretation of Rolle's Mean value theorem.
2. (a) Let $f(x)=e^x$ and $g(x)=e^{-x}$ .Use Cauchy's mean value theorem to show that 'c' is the arithmetic mean between a and b.

   (b) Prove that,

$$\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}, if\ a<b$$

<u>CHAPTER 5 </u>: SUCCESSIVE DIFFERENTIATION

2 marks

1. Find n^{th} derivative of a^{mx}.
2. Find 100^{th} derivative of $(ax + b)^{100}$.
3. Find n^{th} derivative of the following function:

$$y=\frac{1}{x^2-4x+3}$$

4. If $y=e^{2x}.\sin(x+5)$ then find y_n.
5. If $y=e^{3x}.\cos(4x+5)$ then find y_n.
6. Find the 15^{th} derivative of the function:

$$y=\sin(3x-2)$$

4 marks

1. Find the nth derivative of the function:
   $$y = \sin 3x \cos 2x$$
2. Evaluate:
   $$\lim_{x \to 1}\left(\frac{1}{x-1} - \frac{1}{\log x}\right)$$
3. If $y = e^{\tan^{-1} x}$ then prove that $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$
4. If $y = \sin^{-1} x$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$
5. Evaluate:
   $$\lim_{x \to \infty}\left(\frac{1}{x}\right)^{\frac{1}{x}}$$

8 marks

1. State Liebnitz's theorem-
   If $y = \tan^{-1} x$ then prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$
2. State and prove Liebnitz's theorem for the n^{th} derivative of product of two functions.

Miscellaneous Problems:

Q1: Find the n^{th} derivative of the following functions:

1) $y = e^{2x}.\sin(x+5)$
2) $y = \log(3-2x)$
3) $y = \dfrac{1}{x^2 - 5x + 6}$
4) $y = \log(2x-5)$
5) $y = \dfrac{x^2}{(x+2)(2x+3)}$

Q2: Solve:

1) Find y_n for $y = x^3 \sin x$
2) If $y = \sin(m \sin^{-1} x)$,then prove that:
   $$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n$$
3) Find y_n for $y = x^2 \cos(2x-3)$
4) Derive the expression for n^{th} derivative of $e^{ax}\cos(bx+c)$

CHAPTER 6: TAYLOR'S AND MACLAURIN'S THEOREM

2 marks

1. By Maclaurin's series expansion expand real valued function:
   $$f(x) = \cos 3x$$
2. State Taylor's theorem with Lagrange's form of remainder.

4 marks

1. Expand $x^4 - 5x^3 + x^2 - 3x + 4$ in the powers of (x - 1).
2. Assuming validity, obtain expansion of $f(x) = \sec x$ by Maclaurin's theorem upto x^4
3. Expand $3x^3 - 2x^2 + x - 5$ in the powers of (x - 3) by using Taylor series expansion.
4. Assuming validity of expansion prove that:
   $$e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30}.........$$
5. Assuming validity of expansion, prove that:
   $$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - ....$$
6. Expand $x^3 + 3x^2 - x + 2$ in ascending powers of (x - 2).
7. Using Taylor's theorem show that:
   $$\lim_{x\to 0} \frac{e^x - (1 + \sin x)}{x^2} = \frac{1}{2}$$

8 marks

1. State Maclaurin's theorem and use it to find power series expansion of sin2x.
2. (a) Assuming validity of the expansion, expand
   $$\log \sqrt{\frac{1+x}{1-x}}$$
   (b) Expand $x^3 - 4x + 1$ in ascending powers of (x – 1).
3. State Maclaurin's theorem with Cauchy's form of remainder.

   Expand:
   $$x^3 + 3x^2 - x + 2$$
   In ascending powers of (x – 2).

CHAPTER 7: MATRICES AND SYSTEM OF LINEAR EQUATIONS

2 marks

1. Find rank of A where $A=\begin{bmatrix}1 & 2 & -3\\ 2 & -5 & 9\\ 3 & -2 & 1\end{bmatrix}$
2. Write the solution set for the system
   $x+y-2z-w=0$
3. Reduce the matrix A to echelon form:
   $$A=\begin{bmatrix}1 & 5 & 7\\ 11 & 2 & 0\\ 0 & 0 & -5\end{bmatrix}$$
4. Consider the following system of linear equations:
   $$ax+y=0, \quad x-y=0$$
   For which value of 'a' the system will have unique solution?
5. If order of the matrix A is 3x5, then what are the possible maximum values of row rank and column rank for A.
6. Test whether the following matrix is in reduced row echelon form. Justify.
   $$\begin{bmatrix}1 & 0 & -1 & 2\\ 0 & 1 & 3 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

4 marks

1. Solve the following system by Gauss - elimination method:
   $$x-y-z+2w=1$$
   $$2x-y+4z+w=1$$
   $$3x+y-5z+4w=-3$$
2. Solve the following system by Gauss – elimination method:
   $$x-y+z=5$$
   $$9x+3y+z=1$$
   $$x+y+z=-1$$
3. Find column rank of the matrix A where:
   $$A=\begin{bmatrix}1 & 1 & 0 & -1\\ 1 & 2 & 3 & 0\\ 2 & 3 & 3 & -1\end{bmatrix}$$

4. Solve by Gauss elimination method:

$$x + y + 2 = 0$$
$$y + z = 1$$
$$x + z = 1$$

8 marks

1. Find LU – Factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 5 & -9 \\ 3 & -2 & 3 \end{bmatrix}$$

And use it to solve the system of linear equations

$$x + 2y - 4z = -4$$
$$2x + 5y - 9z = -10$$
$$3x - 2y + 3z = 11$$

2. Using LU decomposition, solve the following system:

$$\begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -22 \\ 3 \end{bmatrix}$$

3. Using LU decomposition, solve the following system:

$$\begin{bmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$

****